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## Optimum Pole Shape for Sextupole Magnet Having Circular Pole Tips

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Harmonic field components in sextupole magnets having circular pole tips are numerically obtained by using a two dimensional scalar potential under assumed boundary conditions. Optimum radii of pole tips are found for various depths of coil slots.

### I. INTRODUCTION

In the high resolution momentum analyzer system for particle beam, dipole, and quadrupole magnets are used to analyze the momentum of beam and to converge beam flux respectively. In addition to their fundamental functions, the dipole and quadrupole magnets also cause undesirable higher-order aberrations. The aberration often becomes a serious problem when a high resolution is required for the analyzer system. The second order aberration which affects significantly the resolution can be eliminated or at least reduced by employing sextupole magnets at appropriate locations in the beam optical system. It is desirable for these sextupole magnets that the sextupole component is dominant by suppressing the higher order components to as small as possible amount.

For the above purpose, the profile of the pole piece must be a part of a curve of a hyperbola in order to assure rigorous  $r^2$  proportionality of the magnetic field up to the vicinity of pole surface, where  $r$  is the radial distance from the center axis of the magnet. However, precise fabrication of hyperbolic pole tips is rather difficult. The circular pole tips are simple to fabricate in place of hyperbolic ones. It is then important to estimate the harmonic field contents for magnets having circular pole tips in order to determine the optimum pole shape. This paper gives a numerical calculation of harmonic field contents for sextupole magnets having circular pole tips and discusses the optimum pole shape designs.

Calculations of magnetic field have been carried out for several pole shapes by using conformal transformations. Sonoda *et al.*<sup>1)</sup> treated the magnetic field for a trapezoidal pole profile and showed that the deviation from  $r^2$  proportionality is negligibly small for  $r < 0.6r_0$  where  $r_0$  is the minimum distance of the pole tip from the center. Audoin<sup>2)</sup> calculated the magnetic field for the case where the poles show a cross section, whose profile consists of an arc of a circle tangential to the sides of an angle whose apex is on the axis of the lens. These pole shapes seem not necessarily adequate for the correction lens of the second order aberration, because the higher order harmonic components

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than sextupole should be suppressed up to the radius as large as possible.

In this paper, the harmonic field contents are calculated for a pole shape as shown in Fig. 1 by using a formulation of two dimensional scalar potential as given by one of the authors (M. K.)<sup>3)</sup> and Bellendir and Lari<sup>4)</sup> for quadrupole magnet. The optimum radii of circular pole tips are searched for various locations of coil,  $L_{cut}$ . Calculations for a modified coil shape as shown in Fig. 2 are also given.

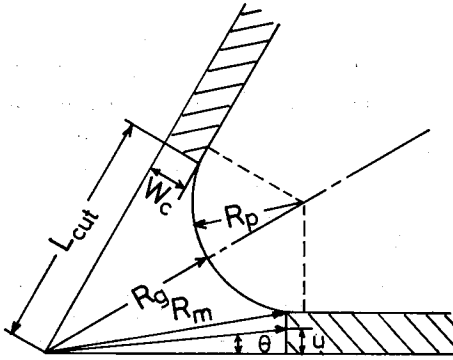


Fig. 1. Circular pole shape with the coil truncation at an arbitrary distance  $L_{cut}$ . Shaded areas show coil space.

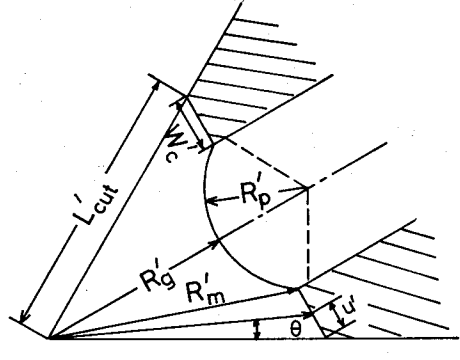


Fig. 2. The same as in Fig. 1 except for a different coil. The coil edge lies on a line perpendicular to the symmetry axis of pole.

## II. METHOD OF CALCULATION

In the region of zero current density, the scalar magnetic potential,  $\Psi$ , should satisfy the Laplace's equation

$$\nabla^2 \Psi(r, \theta) = 0. \quad (1)$$

The general solution of Eq. (1) is given in the polar co-ordinate  $(r, \theta)$  by

$$\Psi(r, \theta) = \sum_{n=1}^{\infty} (A_n \sin n\theta + B_n \cos n\theta) \times (C_n r^n + D_n r^{-n}) + (E + F\theta)(G + H \ln r), \quad (2)$$

where  $A \sim H$  are arbitrary constants. Boundary conditions and symmetry conditions for symmetric sextupole magnet are given as follows:

- (1)  $2\pi$  periodicity in  $\theta$ ,
- (2) finiteness at the origin,
- (3)  $\pi/3$  antisymmetry,
- (4) mirror symmetry with respect to  $\theta = \pi/6$ .

Applying these conditions to Eq. (2), the general solution is reduced to

$$\Psi(r, \theta) = \sum_{n=3, 9, 15, \dots}^{\infty} A_n r^n \sin n\theta. \quad (3)$$

Radial and azimuthal components of magnetic field are

$$B_r(r, \theta) = -\frac{\partial \Psi}{\partial r} = -\sum_{n=3,9,15,\dots}^{\infty} n A_n r^{n-1} \sin n\theta, \quad (4)$$

$$B_\theta(r, \theta) = -\frac{1}{r} \frac{\partial \Psi}{\partial \theta} = -\sum_{n=3,9,15,\dots}^{\infty} n A_n r^{n-1} \cos n\theta. \quad (5)$$

If the potential  $\phi_i$  is given at  $M$  points, the coefficients  $A_n$  can be determined by fitting  $\Psi(r, \theta)$  to  $\phi_i$ 's as follows:

$$\partial \sigma / \partial A_p = 0 \quad (p=3, 9, 15, \dots), \quad (6)$$

$$\sigma = \sum_{i=1}^M (\Psi(r_i, \theta_i) - \phi_i)^2 w_i, \quad (7)$$

where  $w_i$  is the weighting factor for the  $i$ -th point. The coefficients  $A_n$ 's can be found by solving the following  $N$  simultaneous linear equations obtained from Eq. (6),

$$\sum_{n=3,9,15,\dots}^{6N-3} A_n \left( \sum_{i=1}^M \rho_{ni} \rho_{pi} \right) = \sum_{i=1}^M \phi_i \rho_{pi}, \quad (p=3, 9, 15, \dots, 6N-3), \quad (8)$$

where  $\rho_{pi} = r_i^p \sin p\theta$ . Here, higher harmonic components than the  $(6N-3)$ -th are neglected from the summation. The strength of the  $n$ -th harmonic field components at the radius of the pole gap is given by

$$(B_n)_{R_g} = n A_n R_g^{n-1}. \quad (9)$$

The potential is assumed to be constant on the pole surface and to decrease linearly along the coil edge to zero on the axis. That is, the potential can be put as follows:

$$\phi_i = \begin{cases} 1, & \text{on the pole tip,} \\ u/W_c \text{ (or } u'/W'_c), & \text{on the coil edge,} \end{cases}$$

where the notations are given in Figs. 1 and 2. The widths of coils  $W_c$  (for the pole shape of Fig. 1) and  $W'_c$  (Fig. 2) are given as follows:

$$W_c = (R_g + R_p) \sin \frac{\pi}{6} - \left\{ R_p^2 - \left[ (R_g + R_p) \cos \frac{\pi}{6} - L_{\text{cut}} \right]^2 \right\}^{1/2}, \quad (10)$$

$$W'_c = L'_{\text{cut}} \sin \frac{\pi}{6} - \left\{ R_p'^2 - \left[ (R'_g + R'_p) - L'_{\text{cut}} \cos \frac{\pi}{6} \right]^2 \right\}^{1/2}. \quad (11)$$

The present calculations were carried out over the angular range from  $0^\circ$  to  $60^\circ$  for symmetric sextupole magnets. The potentials were given at 100 points with the constant angular step  $\Delta\theta = \pi/300$ . The weighting factor  $w_i$  was set to be unity for simplicity. The maximum number of harmonic components  $N$  was taken to be 8. Numerical calculations were carried out by using the double precision modes of electronic computers, TOSBAC-3400 (96 bits) and HITAC-5020E (64 bits).

### III. RESULTS

The ratio of each harmonic field component to the sextupole one,  $B_n/B_3$ , for the pole shape of Fig. 1 is shown in Figs. 3~6 as a function of  $R_p/R_g$  for various values of

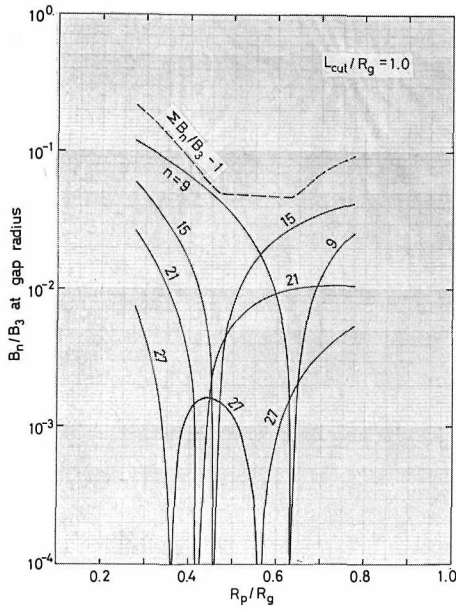


Fig. 3. Harmonic field contents for circular poles given in Fig. 1. The value of  $n$  in the sum  $\Sigma B_n/B_3 - 1$  is from 3 to 45 in steps of 6. The higher harmonics than  $n=27$  are almost less than  $10^{-4}$ .

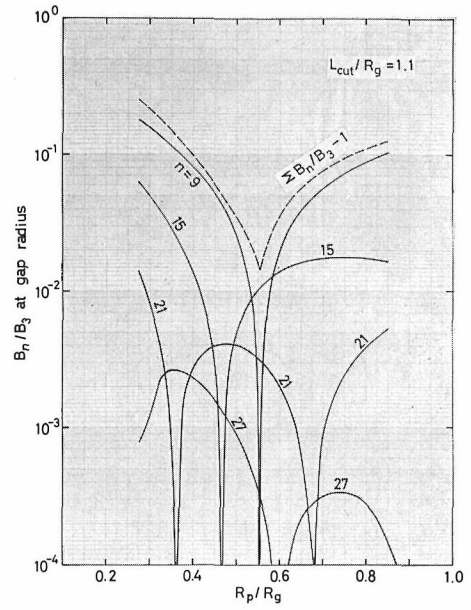


Fig. 4. The same as in Fig. 3. except for different values of  $L_{cut}$ .

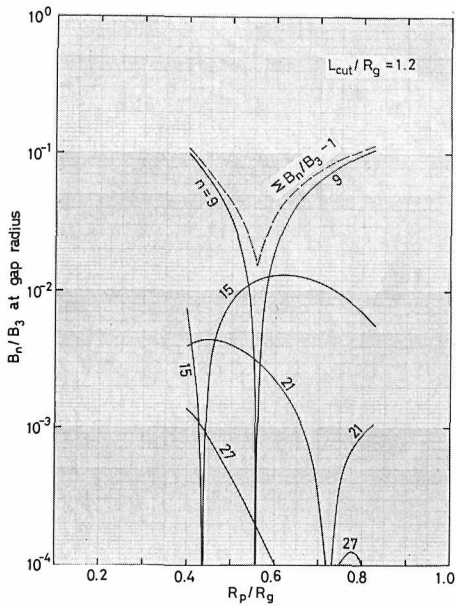


Fig. 5. The same as in Fig. 3 except for different values of  $L_{cut}$ .

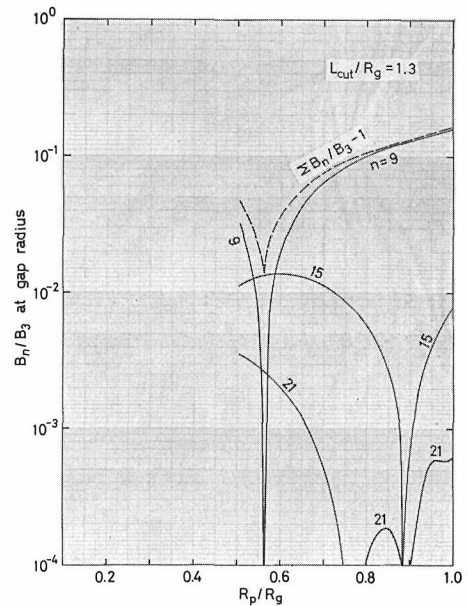


Fig. 6. The same as in Fig. 3 except for different values of  $L_{cut}$ .

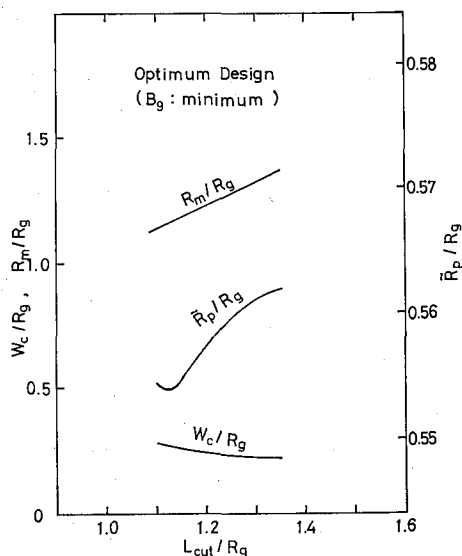


Fig. 7. Optimum radius  $\tilde{R}_p$  of circular pole tip as a function of  $L_{cut}/R_g$ . Corresponding values of  $W_c/R_g$  and  $R_m/R_g$  are also shown.

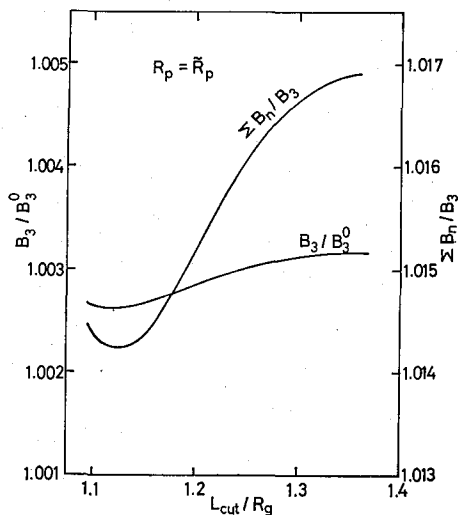


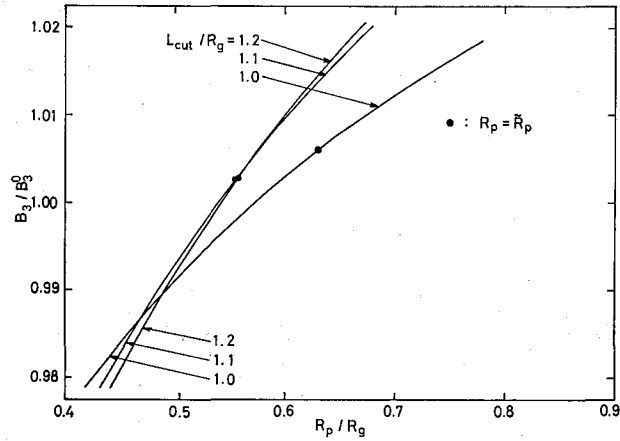
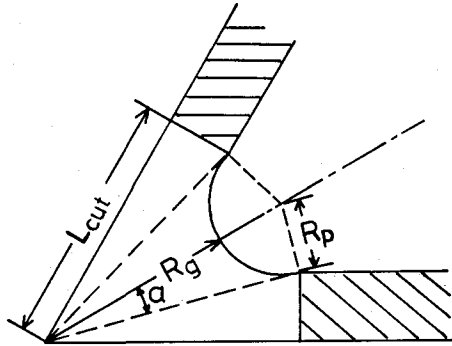
Fig. 8. Sum of harmonic field contents and the ratio of  $B_3$  to  $B_3^0$ , as a function of  $L_{cut}/R_g$  for the case of optimum pole tip design ( $R_p = \tilde{R}_p$ ).

$L_{cut}/R_g$ . The sums of higher components are also shown. Figure 7 gives  $\tilde{R}_p/R_g$ ,  $W_c/R_g$  and  $R_m/R_g$  as a function of  $L_{cut}/R_g$ . Here the optimum radius of circular pole tip,  $\tilde{R}_p$ , is defined as the radius which minimizes the component  $B_9$ . These radii are as follows,  $\tilde{R}_p/R_g = 0.5544$  (for  $L_{cut}/R_g = 1.10$ ), 0.5547 (1.15), 0.5573 (1.20), 0.5596 (1.25), 0.5611 (1.30), and 0.5619 (1.35) within an accuracy of  $\pm 0.0001$ . From the results presented in these figures we can conclude as follows:

- (1) For  $L_{cut}/R_g$  larger than or as large as 1.1, the main higher harmonic component is  $B_9$ . Consequently both  $B_9$  and the sum  $B_n/B_3$  take their own minima for nearly the same value of  $R_p$ . On the other hand, for  $L_{cut}/R_g = 1.0$ ,  $B_{15}$  is comparable with  $B_9$  and the sum  $B_n/B_3$  shows no sharp minimum.
- (2) The magnitude of  $B_9/B_3$  depends significantly on  $R_p/R_g$  while not so significantly on  $L_{cut}/R_g$ .
- (3) The optimum radius  $\tilde{R}_p/R_g$  is not so sensitive to  $L_{cut}/R_g$ .

Figure 8 shows the sum of harmonic field components as a function of  $L_{cut}/R_g$  for the pole profile having the optimum pole tip radius. The total deviation from the sextupole field,  $\Sigma B_n/B_3 - 1$ , is less than 1.7% and has a minimum at  $L_{cut}/R_g \sim 1.13$ . The deviation does not seriously depend on  $L_{cut}/R_g$ . Generally, it is expected for the optimum pole shape design that  $B_3$  is very close to  $B_3^0$  which is the flux density expected in an ideal sextupole magnet having a pole shape of a hyperbola. The magnitude of  $B_3/B_3^0$  is nearly equal to unity as shown in the figure. Figure 9 gives  $B_3/B_3^0$  for various  $L_{cut}/R_g$  as a function of  $R_p/R_g$ .

The results of present calculations are compared with those of Audoin.<sup>2)</sup> Comparison is made for a geometry as shown in Fig. 10. The obtained ratio  $B_9/B_3$  at the gap radius are shown in Table I for the pole shapes with various angles  $\alpha$ . The values

Fig. 9. The ratio of  $B_3$  to  $B_3^0$ , as a function of  $R_p/R_g$  for various  $L_{cut}$ 's.Fig. 10. Circular pole shape chosen to compare the results of Audoin<sup>2)</sup> with ours. Shaded areas show coil space.Table I. Comparison of the Ratios of  $B_3/B_3$  at Gap Radius for Present Calculations and Audoin's Ones.

$\alpha$	$R_p/R_g$	$L_{cut}/R_g$	present $(B_3/B_3)_{R_g}$	Audoin
10°	0.21014	1.11988	0.2644	0.2943
12.5°	0.27623	1.18832	0.2121	0.2187
15°	0.34920	1.25883	0.1479	0.1490
17.5°	0.43002	1.33152	0.0835	0.1088
20°	0.51980	1.40645	0.0243	0.0610

of  $(B_3/B_3)_{R_g}$  in the two calculations nearly agree with each other. The small discrepancy will be due to a different geometry of Audoin from ours. In the case of Audoin, the pole shape slightly differs from circle and, moreover, the coil effect is not taken into account.

Harmonic field contents calculated for another pole shape of Fig. 2 are presented in Figs. 11 and 12 for  $L'_{cut}/R'_g = 1.34$  and 1.44, respectively. The optimum pole tip

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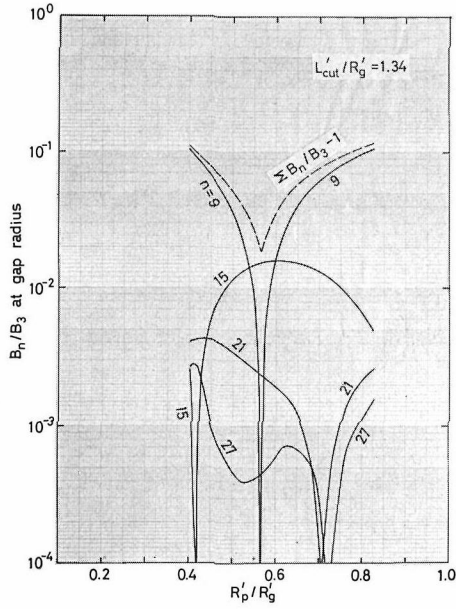


Fig. 11. Harmonic field contents for circular poles given in Fig. 2.

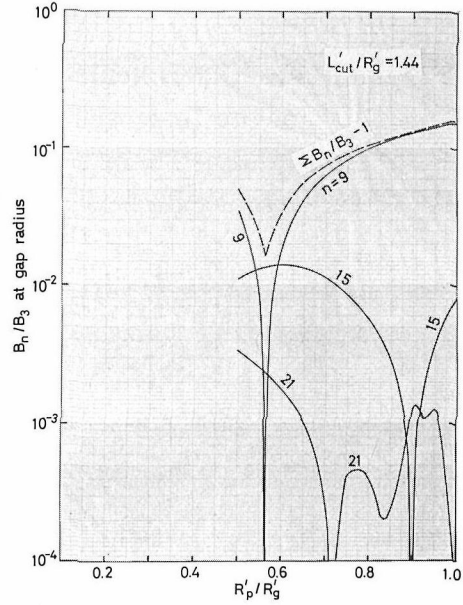


Fig. 12. The same as in Fig. 11 except for a different value of  $L'_{cut}$ .

radii are as follows,  $R'_p/R'_g = 0.5651$  (for  $L'_{cut}/R'_g = 1.34$ ) and  $0.5623$  ( $1.44$ ) within an accuracy of  $\pm 0.0001$ .

From the present calculations, we can accurately obtain the optimum pole tip radius for a sextupole magnet, if the pole gap and the coil position are once fixed. In the coil design,  $L_{cut}/R_g$  should be larger than or as large as  $1.1$ , because much smaller  $L_{cut}/R_g$  gives considerably large amount of undesirable higher harmonic contents almost irrespectively of  $R_p/R_g$ .

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